Problem set #1
Monetary Theory
ECON 530
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I. IS-LM model with monetary policy implications

1. The optimal path of consumption in a dynamic model:

a. Consider the following first-order differential equation:

$$\dot{A}(t) = r\,A(t) + Y(t) - C(t) \qquad (1)$$

where A stands for assets, and Y denotes labor income. We assume that the real interest rate (r) is constant, and C denotes consumption. Use the transversality condition (for example, the condition that states that the present discounted value of assets (A) goes to zero as time approaches infinity) to derive a solution for lifetime consumption.

b. Consider the following lifetime utility function

$$V = \int_0^{\infty} e^{-\beta t}\, ln\, C(t)\, dt \qquad (2)$$

where ln stands for the natural log and β denotes the time preference rate. From class notes, use the Hamiltonian approach, rely on the dynamic budget constraint expressed in equation (1) to maximize the lifetime utility function expressed in equation (2) in order to obtain the Euler equation.

2. Understanding the role of monetary policy in a simple Keynesian model:

a. Consider the following IS-LM model:

$$\begin{aligned} Y &= C + I + G \\ C &= C(Y) \qquad 0 < C_Y < 1 \\ I &= I(i) \qquad I_i < 0 \\ \frac{M}{P} &= m(Y, i) \qquad m_y > 0 \; and \; m_i < 0 \end{aligned}$$

where in equilibrium: total expenditure in the economy is equal to total output (Y). The sum of private consumption expenditures (C), private investment expenditures

(I), and final government expenditures (G) denote total expenditures in the economy.

(i). Assume that prices are fixed in the short run and the central bank targets the money supply (for example, the case where real money balances are exogenous, but the equilibrium interest rate and output are endogenous). Firstly, find the effects of an exogenous government spending shock on the equilibrium interest rate and output (use Cramer's rule); then show the adjustment in the economy by using an IS-LM diagram (you must derive and discuss the slopes of IS and LM curves). Discuss why the effect of fiscal policy is subject to a crowding-out effect in this monetary policy regime (derive this specific crowding effect. How does the interest-rate elasticity of money demand impact the crowding-out effect and consequently the spending multiplier?).

(ii). Now assume that both prices (P) and output (Y) are endogenous. Total differentiate the IS relation and the LM equation; then derive the aggregate demand curve. Discuss the effect of monetary policy in the liquidity trap (you should know what this means when the economy is in the liquidity trap, don't forget to discuss the shape of the money demand curve). Show that the AD curve is vertical (independent of the price level) in the liquidity trap and discuss the implications for the equilibrium determination of the level of output in such an environment. More importantly, how can policymakers move the economy back towards full employment when monetary policy is impotent (think about Keynes's argument).

II. Business cycle, the Phillips curve and monetary policy

1. The price setting relation is given by

$$P_t = (1 + m)W_t \qquad (1)$$

where W denotes the nominal wage P is the average price level and m is the mark-up (for simplicity, assume that m is relatively small). The wage setting relation is given by

$$W_t = P_t^e(1 - \varphi U_t) \qquad (2)$$

where φ denotes the effect of unemployment on wages. Here, the nominal wage depends on the expected price level (P^e) and the unemployment rate (U_t). Firstly, derive the expectation augmented Phillips curve and the non-accelerating inflation rate of unemployment (U_n). Secondly, assuming that expectations about inflation are formed according to the following equation

$$\pi_t^e = \pi_{t-1} \qquad (3)$$

Then rely on equation (3) and the NAIRU to derive the expectations-augmented Phillips curve which shows a relation between the unemployment rate, the natural

unemployment rate (U_n), and the change in the inflation rate. Describe the trade-off illustrated by the expectation augmented Phillips curve, which the central bank can exploit in the short run. How does the new Keynesian Phillips curve differ from the expectation augmented Phillips curve derived here (discuss briefly).